Problem Sheet 4

Problems in Part A will be discussed in class. Problems in Part B come with solutions and should be tried at home.

Part A

(4.1) For the following linear programming problems,

$$\begin{aligned} \text{maximize} \quad & x_1 + 2x_2 \\ \text{subject to} \quad & x_1 + x_2 \leq 2 \\ & x_1 - x_2 \leq 1 \\ & x_1 \geq -1 \end{aligned} \tag{LP1}$$

$$\begin{aligned} \text{maximize} \quad & x_1 + x_2 \\ \text{subject to} \quad & x_2 - x_1 \leq 2 \\ & x_1 + x_2 \leq 8 \\ & x_1 + 2x_2 \leq 10 \\ & x_1 \leq 4 \\ & x_1 \geq 0 \\ & x_2 \geq 0. \end{aligned} \tag{LP2}$$

(a) Sketch the polyhedron of feasible points and find the vertices (you can do this based on the diagram, but also by computation if you like. To determinte the vertices by computation, consider all subsets of two equations and solve these 2×2 systems with equality instead of inequality. This gives a list of points, and those that satisfy the constraints are the verices.);

(b) Find a solution, if it exists (you can find the solution visually, but you may use a computer program to verify the result).

(4.2) Show that there exists a vector $\boldsymbol{x} \neq \boldsymbol{0}$ satisfying

$$\boldsymbol{x} \geq 0, \quad \boldsymbol{A}\boldsymbol{x} = \boldsymbol{0} \tag{P}$$

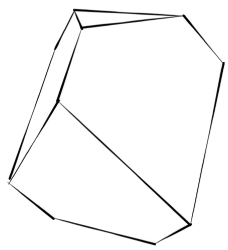
if and only if there is no vector $\boldsymbol{y}$ such that

$$\boldsymbol{A}^\top \boldsymbol{y} > \boldsymbol{0}. \tag{D}$$

Give a geometric interpretation of this fact.

Part B

(4.3) Consider the following polyhedron from Example 8.5, Lecture 8.



$$\begin{aligned}
0.7071x_1 - 0.4082x_2 + 0.3773x_3 &\le 1 \quad (1)\\
-0.7071x_1 + 0.4082x_2 - 0.3773x_3 &\le 1 \quad (2)\\
0.7071x_1 + 0.4082x_2 - 0.3773x_3 &\le 1 \quad (3)\\
-0.7071x_1 - 0.4082x_2 + 0.3773x_3 &\le 1 \quad (4)\\
0.8165x_2 + 0.3773x_3 &\le 1 \quad (5)\\
-0.8165x_2 - 0.3773x_3 &\le 1 \quad (6)\\
0.6313x_3 &\le 1 \quad (7)\\
-0.6313x_3 &\le 1 \quad (8)
\end{aligned}$$

Figure 1: Truncated triangular trapezohedron and defining equations

(a) Print the foldout in Figure 2 and assemble the polyhedron.

(b) On the foldout, label each of the eight faces with the number of a corresponding equation (due to symmetry, such an assignment is not unique).

(c) Each of the 12 vertices arises as the intersection of three affine hyperplanes, given as the set of points where three of the equations in Figure 1 are equalities. Using a computing system such as Python or MATLAB, determine the vertices.

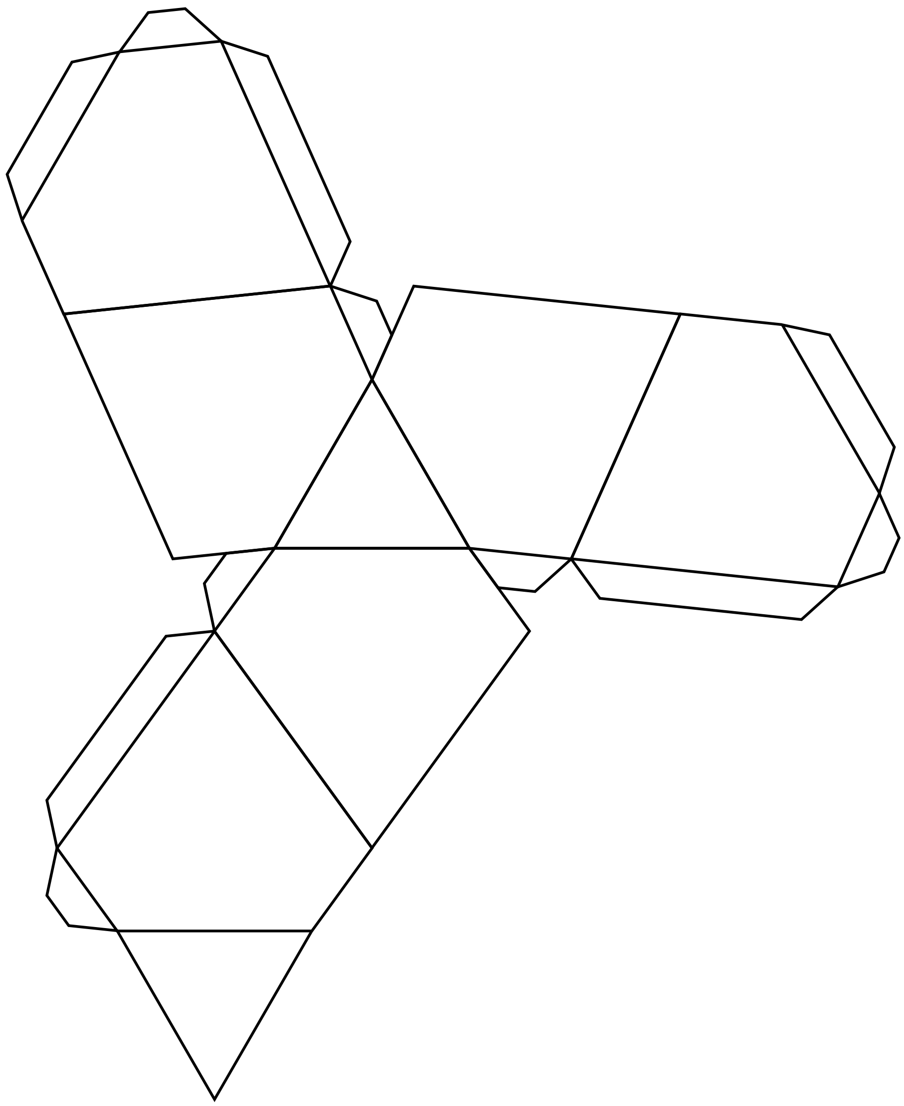


Figure 2: Foldout of a polyhedron