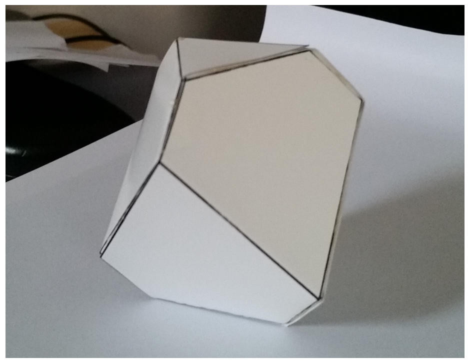
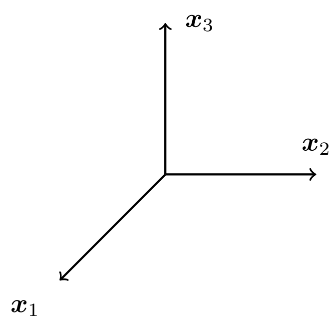
Solutions to Part B of Problem Sheet 4

Solution (4.3)

(a) This is the solution:



(b) Notice that the equations come in positive/negative pairs: each such pair corresponds to opposite faces. The equations (7) and (8) correspond to the triangles, as they give conditions only on the x_3 axis. Let's fix a coordinate system oriented as follows (we only care about the orientation, not the absolute positioning!) Then



Equation (7) corresponds to the top triangle, (8) on the bottom triangle, Equations (1), (4), (5) describe the upward-facing superman shapes, and (2), (3) and (6) the downward facing ones (because their x_3-coordinate is negative). Taking one of the upward facing shapes as being parallel to the x_1 axis, this corresponds to Equation (5), because of the x_1 term missing there. The upward facing shape to the left of it is then (1) (because of the positive x_1 coordinate), and the other one (4). In summary, the following labels are consistent.

(c) From the above diagram (or better, the assembled polyhedron) we see precisely which hyperplanes intersect to create the vertices:

$$\begin{aligned}&(157), (457), (147), (146), (245), (135),\\&(136), (246), (235), (248), (238), (368)\end{aligned} \tag{2}$$

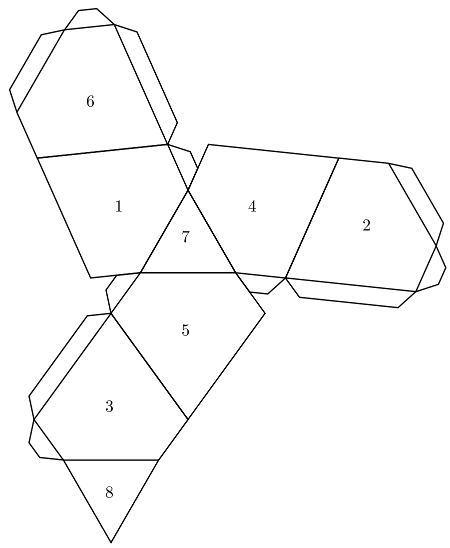


Figure 1: Foldout with supporting hyperplanes

From this list we get a recipe for determining the vertices from the system of inequalities: for the triple (157) select the equations 1, 5 and 7, and solve the corresponding system of three equations in three unknowns. The solution will be the vertex. Implementing this in MATLAB, we get the vertices

$$\boldsymbol{v}_1 = \begin{pmatrix} -1.4142 \\ -0.8165 \\ -0.8835 \end{pmatrix}, \boldsymbol{v}_2 = \begin{pmatrix} -1.4142 \\ 0.8165 \\ 0.8835 \end{pmatrix}, \boldsymbol{v}_3 = \begin{pmatrix} -0.8536 \\ -0.4928 \\ -1.5840 \end{pmatrix}, \boldsymbol{v}_4 = \begin{pmatrix} -0.8536 \\ 0.4928 \\ 1.5840 \end{pmatrix},$$
$$\boldsymbol{v}_5 = \begin{pmatrix} -0.0000 \\ -1.6330 \\ 0.8835 \end{pmatrix}, \boldsymbol{v}_6 = \begin{pmatrix} 0.0000 \\ -0.9856 \\ 1.5840 \end{pmatrix}, \boldsymbol{v}_7 = \begin{pmatrix} -0.0000 \\ 0.9856 \\ -1.5840 \end{pmatrix}, \boldsymbol{v}_8 = \begin{pmatrix} 0.0000 \\ 1.6330 \\ -0.8835 \end{pmatrix},$$
$$\boldsymbol{v}_9 = \begin{pmatrix} 0.8536 \\ -0.4928 \\ -1.5840 \end{pmatrix}, \boldsymbol{v}_{10} = \begin{pmatrix} 0.8536 \\ 0.4928 \\ 1.5840 \end{pmatrix}, \boldsymbol{v}_{11} = \begin{pmatrix} 1.4142 \\ -0.8165 \\ -0.8835 \end{pmatrix}, \boldsymbol{v}_{12} = \begin{pmatrix} 1.4142 \\ 0.8165 \\ 0.8835 \end{pmatrix}.$$